

# Fracture functions in the very forward limit

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This talk gives a brief discussion of extended fracture functions, which parametrise the non-perturbative physics in the target fragmentation region of semi-inclusive DIS. In the forward limit  $z \rightarrow 1$ , it can be seen that fracture functions can be identified with insertions of composite operators. This enables polarised fracture functions to be used to test a target-independence hypothesis of the “proton spin effect”.

## 1. Introduction

The “proton spin effect” [1] is the anomalous suppression of the Ellis-Jaffe sum-rule for the first moment of  $g_1(x, Q^2)$  in inclusive Deep Inelastic Scattering (DIS). It can be understood in terms of a target-independent topological charge screening mechanism [2]. Central to this analysis is the need to express the moments of the target distribution functions as matrix elements of local, composite operators which appear in the operator product expansion (OPE).

In a recent proposal [3], a suggestion was made to test the target-independence hypothesis in the target fragmentation region of semi-inclusive DIS where a hadron is tagged in the final state. This process factorises at large  $Q^2$  as follows:

Semi-inclusive structure function =  
(Hard physics)  $\otimes$  (Fracture function).

Unlike parton densities in inclusive DIS, fracture functions cannot be related to Green functions of local, composite operators. This prevents the analysis of Refs. [2] to be applied rigorously to polarised, semi-inclusive DIS. Instead, the moments of fracture function are represented as generalised, space-like cut vertices [4]. However, in the forward limit,  $z \rightarrow 1$ , where the tagged final state hadron carries most of the nucleon momentum, it can be shown [5] that these cut vertices reduce to objects depending on insertions of local, composite operators. This talk summarises

the main arguments.

## 2. Matrix elements and proper vertices

This section summarises the explanation of the EMC/SMC “proton spin” effect as one of topological charge screening [2]. The important point relevant to this analysis is that the target distribution functions must be expressed in terms of matrix elements of local, composite operators.

Measurements by the EMC and SMC collaborations [1] of the first moment  $\Gamma_1^p$  of the polarised structure function  $g_1(x, Q^2)$  have found it to be suppressed compared with its OZI expectation. The OPE for  $\Gamma_1^p$  is

$$\begin{aligned}\Gamma_1^p &= \int_0^1 dx g_1^p(x, Q^2) \\ &= \frac{1}{12} C_1^{\text{NS}}(\alpha_s) \left( a^3 + \frac{1}{3} a^8 \right) \\ &+ \frac{1}{9} C_1^{\text{S}}(\alpha_s) a^0(Q^2),\end{aligned}\tag{1}$$

where the axial charges  $a^i$  are defined as reduced matrix elements:

$$\begin{aligned}\frac{1}{2} a^3 s_\mu &= \langle p; s | A_\mu^3 | p; s \rangle \\ \frac{1}{2\sqrt{3}} a^8 s_\mu &= \langle p; s | A_\mu^8 | p; s \rangle \\ a^0(Q^2) &= \langle p; s | A_\mu^0 | p; s \rangle\end{aligned}\tag{2}$$

Note that the flavour singlet  $a^0$  is scale-dependent because of the axial anomaly. In the QCD parton model [6],

$$a^3 = \Delta u - \Delta d$$

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$$\begin{aligned}
a^8 &= \Delta u + \Delta d - 2\Delta s \\
a^0(Q^2) &= \Delta u + \Delta d + \Delta s - n_f \frac{\alpha_s}{2\pi} \Delta g(Q^2) \quad (3)
\end{aligned}$$

On the assumption of the OZI rule,  $\Delta s \simeq 0 \simeq \Delta g$ ,  $a^0 \simeq a^8$ , but experimentally  $a^0$  is found to be suppressed compared with  $a^8$ . This is not surprising given the scale-dependence of  $a^0$ ; any explanation of the suppression should take account of this.

One such explanation, given in a series of papers by Narison, Shore and Veneziano [2], is to identify the mechanism with topological charge screening. Using the anomalous Ward identity

$$\partial_\mu A_0^\mu - 2n_f Q \simeq 0, \quad (4)$$

with  $Q = \frac{\alpha_s}{8\pi} \text{tr} G_{\mu\nu} \tilde{G}^{\mu\nu}$  the topological charge density, one can re-express the flavour singlet axial charge as a measure of topological charge:

$$a^0(Q^2) = \frac{1}{2M_p} 2n_f \langle p | Q | p \rangle \quad (5)$$

We can then perform a Legendre transform on the QCD generating functional with respect to  $Q$  and  $\Phi_5 \propto \bar{q}\gamma_5 q$  only; this enables a decomposition of  $a^0$  into products of 1PI vertices and propagators.

$$\begin{aligned}
a^0(Q^2) &= \frac{1}{2M_p} 2n_f [\langle 0 | T(Q Q) | 0 \rangle \Gamma_{Qpp}^{1PI} \\
&\quad + \langle 0 | T(Q \Phi_5) | 0 \rangle \Gamma_{\Phi_5 pp}^{1PI}] \\
&= \frac{1}{2M_p} 2n_f [\chi(0) \Gamma_{Qpp}^{1PI} \\
&\quad + \sqrt{\chi'(0)} \Gamma_{\Phi_5 pp}^{1PI}], \quad (6)
\end{aligned}$$

where

$$\begin{aligned}
\chi(k^2) &= i \int d^d x e^{ik \cdot x} \langle 0 | T(Q(x) Q(0)) | 0 \rangle, \\
\chi'(0) &= \left. \frac{d}{dk^2} \chi(k^2) \right|_{k^2=0}. \quad (7)
\end{aligned}$$

From the chiral Ward identities,  $\chi(0)$  vanishes in the chiral limit. A similar decomposition for  $a^8$  can be made in terms of  $\Gamma_{\Phi_5^8}$ . Assuming [2]  $\Gamma_{\Phi_5^8}$  obeys the OZI rule, it is found that

$$\frac{a^0(Q^2)}{a^8} = \frac{\sqrt{6}}{f_\pi} \sqrt{\chi'(0)}. \quad (8)$$

Thus, the suppression of  $a^0/a^8$  is due to an anomalously small value of the slope of the topological susceptibility  $\chi'(0)$ . Furthermore, this

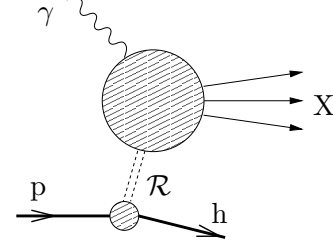


Figure 1. Semi-inclusive DIS in the  $z \rightarrow 1$  limit.

prediction does not depend on the nature of the target.

To test this target-independence hypothesis, it has been proposed in Ref. [3] to measure the polarised structure functions of Regge poles in semi-inclusive DIS in the limit  $z \rightarrow 1$  where  $z$  gives the longitudinal momentum fraction of the final state hadron. (Fig. 1.) In this limit, the polarised semi-inclusive structure function factorises:

$$\begin{aligned}
\sum_i e_i^2 \Delta M_{h/N}(x, z, t, Q^2) &\xrightarrow{z \rightarrow 1} \\
F(t)(1-z)^{-2\alpha_{\mathcal{R}}(t)} g_1^{\mathcal{R}}\left(\frac{x}{1-z}, t, Q^2\right) \quad (9)
\end{aligned}$$

By taking ratios of  $\Delta M_{h/N}$  for different final state hadrons  $h$  and targets  $N = p, n$ , we can effectively compare  $g_1^{\mathcal{R}}$  for different  $\mathcal{R}$ . For example, the ratio of cross-sections of processes with  $\mathcal{R} = \Delta^\pm$  would be

$$\frac{en \rightarrow e\pi^+ X}{ep \rightarrow e\pi^- X} \simeq \frac{2s-1}{2s+2}, \quad (10)$$

where  $s$  is a universal suppression factor measurable in inclusive DIS:

$$s(Q^2) = \frac{C_1^S(\alpha_s)}{C_1^{NS}(\alpha_s)} \frac{a_0(Q^2)}{a^8} \quad (11)$$

To put these predictions on a rigorous footing, we would need to show that one can perform the OPE – 1PI vertex – propagator analysis sketched above on semi-inclusive structure functions. The rest of this talk shows how this is possible in the  $z \rightarrow 1$  limit.

### 3. Cut vertices and semi-inclusive DIS

A simplified discussion of semi-inclusive DIS can be given in the language of  $(\phi^3)_6$  scalar field theory, which like QCD is asymptotically free. To define a Lorentz-scalar semi-inclusive structure function, we can write

$$W(q, p, p') = \frac{Q^2}{2\pi} \sum_X \int d^d x e^{iq \cdot x} \times \langle p | j(x) | h, X \rangle \langle h, X | j(0) | p \rangle, \quad (12)$$

where  $j(x) = \phi^2(x)$  plays the role of the electromagnetic current.  $q$  is the “photon” momentum, and  $p$  and  $p'$  the proton and hadron  $h$  momenta respectively. The convenient kinematical variables are

$$Q^2 = -q^2, \quad t = (p - p')^2, \quad (13)$$

$$z = \frac{p' \cdot q}{p \cdot q}, \quad x = \frac{Q^2}{2p \cdot q}.$$

We also use  $\bar{x} = x/(1 - z)$ . In the target fragmentation region,  $Q^2$  large,  $|t|/Q^2 \ll 1$ ,  $W$  factorises into a convolution of perturbative Wilson co-efficients  $C$  and *extended* fracture functions  $\mathcal{M}$  [4]:

$$W_{\text{targ}}(Q^2, \bar{x}, z, t) = \int_{\bar{x}}^1 \frac{du}{u} \mathcal{M}(u, z, t, \mu) C(\frac{\bar{x}}{u}, Q^2, \mu), \quad (14)$$

Factorisation theorems have been proved for  $(\phi^3)_6$  theory [7] and QCD [8]. It has also been shown [7] that the moments of extended fracture functions  $\mathcal{M}^j$  can be represented as generalised, space-like, cut vertices: (Fig. 2.)

$$\begin{aligned} \mathcal{M}^j(z, t, \mu) &= \int_0^1 du u^{j-1} \mathcal{M}(u, z, t, \mu) \\ &= (p_+ - p'_+)^j |\Lambda(p, p')|^2 \\ &+ \int \frac{d^d k}{(2\pi)^d} (k_+)^j \theta(0 < k_+ < p_+) \\ &\quad \times \frac{\text{disc}}{(k - p + p')^2} G(k, p, p'), \end{aligned} \quad (15)$$

where

$$\Lambda(p, p') = \Delta_F^{-1}(p) \Delta_F^{-1}(p') \times \langle 0 | T(\phi(p) \phi(-p') \phi(-p + p')) | 0 \rangle, \quad (16)$$

$$G(k, p, p') = \Delta_F^{-2}(p) \Delta_F^{-2}(p') \times \langle 0 | T(\phi_p \phi_{-p'} \phi_{-k} \phi_k \phi_{p'} \phi_{-p}) | 0 \rangle. \quad (17)$$

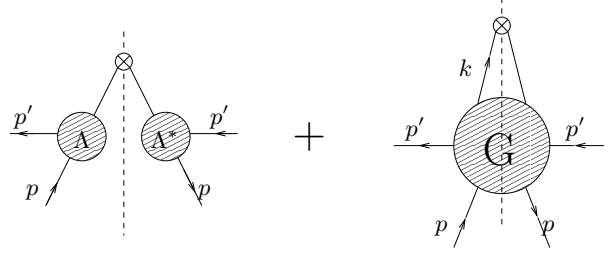


Figure 2. Definition of generalised cut vertex.

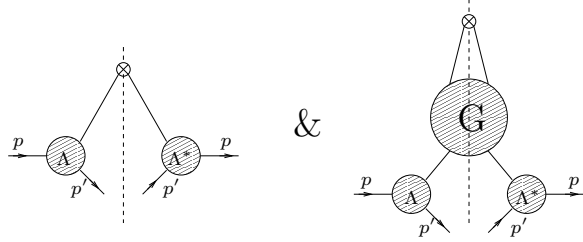


Figure 3. Topology of the dominant one-loop diagrams. Compare equation 18

(The first  $\Lambda$ -term is not required in QCD.)

The moments of fracture functions do not depend on the insertion of a composite operator because of the final-state cut or discontinuity. Unlike in the case of inclusive DIS [9], one cannot remove this final state cut — this is the reason why the OPE is not directly applicable to semi-inclusive DIS. We cannot therefore perform the 1PI vertex – propagator decomposition on them.

However, a one-loop calculation in  $(\phi^3)_6$  theory shows that the class of diagrams contributing to  $\mathcal{M}^j$  which dominate as  $z \rightarrow 1$  have such a simple analytic structure that the final state cut can be removed. Their topology is as in Fig. 3. (For

details, see Ref. [5].) The final result is

$$\begin{aligned} \mathcal{M}^j(z, t) &\xrightarrow{z \rightarrow 1} \left[ (p_+ - p'_+)^j + \int \frac{d^d k}{(2\pi)^d} (k_+)^j G(k, p - p') \right] \Lambda^2(p, p') \\ &= \langle 0 | T(\phi_{p-p'} : \phi(i\partial_+)^j \phi : (0)\phi_{-p+p'}) | 0 \rangle \\ &\quad \times \Delta_F^{-2}(p - p') \Lambda^2(p, p') \end{aligned} \quad (18)$$

where

$$\begin{aligned} G(k_1, k_2) &= \Delta_F^{-2}(k_1) \\ &\quad \times \langle 0 | T(\phi(k_1)\phi(-k_2)\phi(k_2)\phi(-k_1)) | 0 \rangle. \end{aligned} \quad (19)$$

This leading term in  $(1-z)$  depends on the Green function of a composite operator.

The corresponding QCD cut vertices can also be expected to have this property. As  $z \rightarrow 1$ , one can assume that factorisation of hard physics and Regge factorisation [10] are both valid, so that

$$\begin{aligned} \mathcal{M}_{a/hN}^j(z, t) &\longrightarrow \\ &\sum_{\mathcal{R}} F_{\mathcal{R}Nh}(t) (1-z)^{-2\alpha_{\mathcal{R}}(t)} f_{a/\mathcal{R}}^j(t). \end{aligned} \quad (20)$$

If this is the case, then the moments of Reggeon structure functions  $f_{a/\mathcal{R}}^j$  do indeed depend on insertions of composite operators.

Having related fracture functions to insertions of composite operators, one can then perform the 1PI vertex – propagator decomposition on them. In the spin-polarised case, this enables us to use fracture functions as a testing ground for the target-independence hypothesis of the “proton spin effect”.

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